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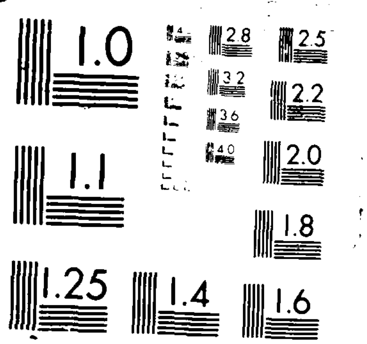
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The focus of this research was primarily feedback stabilization of distributed parameter systems. The principal investigator derived feedback operators for a general calss of distributed systems, which include flexiblem beams, under the constraint of bounded control. Six papers were published, including "Feedback Stabilization in Hilbert Space".

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FINAL PROGRESS REPORT

AFOSR-85-0239

Problems in Nonlinear Continuum Dynamics

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0. Introduction.

The research done under AFOSR-85-0239 falls into three categories:

(1) feedback stabilization of distributed parameter control systems;

(2) continuum dynamics of materials exhibiting phase transitions;

(3) dynamics of reacting chemical systems.

Research in the first two areas has been an ongoing project of the investigator over the past several years. Research in (3) is a new endeavor by the investigator in collaboration with L. A. Segel. The results of the research have and will appear in journal articles. In addition two Ph.D. theses have been completed under the grant and one is currently in progress.

In this report a discussion will be given as to the nature of the research and results obtained in each of the above three categories.

1. Feedback stabilization of distributed parameter control systems.

In this research the investigator concerned himself with finding feedback laws for control systems governed by partial differential equations. In particular those control systems which give the dynamics of aero-elastic systems have been of particular interest. To make the issue more transparent the following example drawn from the work of Dr. J. Hubbard at M.I.T.'s Draper Laboratory may be helpful.

Consider the control system

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^4 w}{\partial x^4} = 0 \quad \text{for} \quad 0 < x < L$$

$$w = \frac{\partial w}{\partial x} = 0 \quad \text{for} \quad x = 0 ,$$

$$\left.\begin{aligned} \frac{\partial^2 w}{\partial x^2} &= -\frac{\partial^3 w}{\partial t^2 \partial x} + f(t) \\ \frac{\partial^3 w}{\partial x^3} &= \frac{\partial^2 w}{\partial t^2} \end{aligned}\right\} \quad x = L \qquad (1.1)$$

Here $w(x,t)$ denotes the displacement of a beam and $f(t)$ is an applied scalar boundary control, $|f(t)| < r$, $r > 0$. In the absence of the control

f the beam oscillates in an almost periodic fashion. The goal is to find a feedback control

$$f(t) = F(w(x,t))$$

where F is some functional of $w(x,t)$ and its derivatives that will drive $w(x,t)$ and $w_t(x,t)$ to zero as $t \to \infty$. This control will then stabilize the system and eliminate undesired oscillations. We note the control law must be nonlinear due to the imposition of the constraint

$$|F(w(x,t))| \leq r \quad .$$

In his research the investigator reformulated this problem in an abstract setting so as to cover a large class of problems in a unified setting. Basically the idea was to place the problem in the format of an evolution equation in a Hilbert space so that (1.1) could be written in the form

$$\frac{du}{dt} = Au + Bf \tag{1.2}$$

where A is the infinitesimal generator of a linear semigroup e^{At} on a Hilbert space H, f lies in a second Hilbert space E and B is a bounded linear operator mapping E to H, u denotes the state of the system. Our constraint is now $\|f\|_E \leq r$.

In this formulation the investigator was able to use the tools of dynamical system theory developed earlier by the investigator, J. Ball, and C. M. Dafermos to find a stabilizing feedback control. The results appear in the paper

"Feedback stabilization of $\frac{du}{dt} = Au + Bf$ in Hilbert space when $\|f\| \leq r$" by M. Slemrod submitted to Mathematics of Control, Signals, and Systems.

It is also interesting to note tht by working in a general setting a problem raised by A. V. Balakrishan for the feedback stabilization of the NASA Spacecraft Control Laboratory Experiment (SCOLE) (see A. V. Balakrishnan, A. Mathematical Formulation of a Large Space Structure Control Problem, 24th Conference on Decision and Control, IEEE, Dec. 1985) was solved in this same paper of the investigator.

2. Continuum dynamics of materials exhibiting phase transitions.

The research done in the area of continuum dynamics of phase transitions centers on understanding dynamics of van der Waals like materials.

The reason for interest in these materials is that they provide the simple mathematical models for materials wh ch can exist in two (or more) phases, e.g. for water: liquid, vapor, solid phase. More exotic materials such as plutonium can exist in even more phases (for plutonium seven phases are possible).

To see analytically what the mathematical issues are recall that the van der Waals constitutive equation for a compressible fluid is given by (see e.g. E. Fermi, Thermodynamics, Dover)

$$p(w,\theta) = \frac{R\theta}{w-b} - \frac{a}{w^2} \tag{2.1}$$

where p is the pressure, θ is the absolute temperature, w is the specific volume, R, a, b are positive constants. It is not hard to show that there is a critical value of θ, the critical temperature θ_{crit}, so that for

$$\theta > \theta_{crit} \quad \frac{\partial p}{\partial w}(w,\theta) < 0 \quad ; \tag{2.2}$$

and for $\theta < \theta_{crit}$

$$\frac{\partial p}{\partial w}(w,\theta) < 0, \qquad b < w < \alpha, \qquad \beta < w < \infty \quad ,$$

where α and β are constants $\alpha < \beta$, and

$$\frac{\partial p}{\partial w}(w,\theta) > 0, \qquad \alpha < w < \beta \ ,$$

(the spinodal region).

The spinodal represents an anamalous region where the usual view that pressure should be monotone decreasing in specific volume is violated. Furthermore substitution of this choice of p into the isothermal balance laws of mass and momentum (in Lagrangian coordinates)

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} p(w) &= 0 \\ \frac{\partial w}{\partial t} - \frac{\partial}{\partial x} u &= 0 \end{aligned} \tag{2.3}$$

where u is the velocity yields a mixed hyperbolic-elliptic initial value problem for $\theta < \theta_{crit}$ i.e. (2.3) is hyperbolic on $b < w < \alpha$, $\beta < w < \infty$ and elliptic on $\alpha < w < \beta$.

The investigator has attempted to understand (2.3) over the past years and written numerous papers on the subject under the present and previous AFOSR grants.

Under the present grant the main work has been on the resolution of the Riemann initial value problem i.e. solvability of the one dimensional initial value problem for a van der Waals fluid (2.3) with initial data

$$\begin{array}{llll} u(x,0) = u_- & & u(x,0) = u_+ & \\ & x < 0 \ ; & & x > 0 \ . \\ w(x,0) = w_- & & w(x,0) = w_+ & \end{array} \tag{2.4}$$

Here u_-, w_-, u_+, w_+ are constants.

Motivated by previous work of the investigator an attempt to solve (2.3), (2.4) was made via the method limiting artificial viscosity. Specifically (2.3) was regularized by the "viscous" system

$$\begin{aligned} u_t + p_x &= \varepsilon t u_{xx} \ , \\ w_t - u_x &= \varepsilon t w_{xx} \ . \end{aligned} \tag{2.5}$$

System (2.5) has the advantage that it admits solutions in the similarity variable $\xi = \frac{x}{t}$, i.e. if we set $u = u(\xi)$, $w = w(\xi)$ and substitute into (2.5) we have

$$\begin{aligned} -\xi u' + p' &= \varepsilon u'' \ , \\ -\xi w' - u' &= \varepsilon w'' \ , \end{aligned} \tag{2.6}$$

and initial conditions (2.4) become

$$u(-\infty) = u_-, \ w(-\infty) = w_-, \ u(+\infty) = u_+, \ w(+\infty) = w_+ \ . \tag{2.7}$$

In his paper

A limiting viscosity approach to the Riemann problem for a van der Waals fluid, to appear <u>Archive for Rational Mechanics and Analysis</u>

the investigator show for initial data in different phases e.g. w_- liquid, w_+ vapor (2.6) always possesses a solution u^ε, w^ε. Also except for one special case the limit as $\varepsilon \to 0+$ of $(u^\varepsilon, w^\varepsilon)$ exists $= (u,w)$ and u, w

satisfy (2.3), (2.4) i.e. the Riemann problem is solved.

In line with this area of research the investigator has supervised two Ph.D. theses at R.P.I.

B. Cassis, The method of compensated compactness applied to a singular perturbed fourth order nonlinear p.d.e. and a mixed hyperbolic-elliptic system of p.d.e.'s., submitted June, 1986.

M. Grinfeld, Topological techniques in dynamic phase transitions, submitted May, 1986.

A third thesis is being written by a student Luis Leon.

3. Dynamics of reacting chemical systems.

This research is a collaboration effort with Professor L. A. Segel of the Weizmann Institute of Science, Rehovot, Israel. The problem is to study an approximation method, usually called the quasi-steady state assumption (QSSA) or pseudo-steady-state hypotheses (PSSH) which arises in the study of biochemical kinetics. In its simplest form, the QSSA deals with kinetics described by systems of ordinary differential equations wherein after an initial fast transient, one (or more) of the dependent variables may be regarded as in steady state with respect to the instantaneous values of the other dependent variables.

The most studied example of the QSSA concerns a biochemical reaction wherein an enzyme (concentration E) reacts reversibly with another chemical (concentration S, the substrate) to form an enzyme-substrate complex (concentration C). After some manipulations and elimination of E the basic problem becomes analysis of the nonlinear system of ordinary differential equations

$$\frac{dS}{dt} = -k_1(E_0 - C) + k_{-1}C \quad , \tag{3.1}$$

$$\frac{dC}{dt} = k_1(E_0 - C)S - (k_{-1} + k_2)C \quad ,$$
$$S(0) = S_0, \ C(0) = 0 \quad , \tag{3.2}$$

where k_{-1}, k_1, k_2 E_0 are positive constants.

In the QSSA it is argued that experimental measurements are generally performed after a relatively short pre-steady period but before the substrate value appreciably decays. It is in this period that chemists argue that C is approximately a constant so that $\frac{dC}{dt} \approx 0$. But if $\frac{dC}{dt} = 0$ then (3.2) tells us

$$C \approx E_0 \ S/(k_m + S) \quad , \quad k_m = \frac{k_{-1} + k_2}{k_1} \quad , \tag{3.3}$$

and by (3.1) S is governed by the estimate

$$\frac{dS}{dt} \approx \frac{-k_2 E_0 S}{(k_m + S)} \quad . \tag{3.4}$$

Approximation (3.3), (3.4) constitute the QSSA.

The result of the research on the QSSA was the following:

(1) L. A. Segel has given an argument based on appropriate choice of non-dimensional parameters to formally suggest when the QSSA holds.

(2) M. Slemrod has used arguments from the qualitative theory of ordinary differential equations to prove Segel's formal conjecture is rigorously true.

The collaboration was a true meshing of two different modes of thought to solve a classical problem in applied mathematics. The authors believe they have the most general result on the validity of the QSSA to date. Furthermore the authors result is stated within the utmost mathematical rigor.

A paper has been prepared for submission to SIAM Review entitled "A Study of the Quasi-steady State Assumption", by L. A. Segel and M. Slemrod.

Publications: AFOSR-85-0239

M. Slemrod, Feedback stabilization of $\frac{du}{dt} = Au + Bf$ in Hilbert space when $\|f\| \leq r$, submitted to Mathematics of Control, Signals, and Systems.

M. Slemrod, A limiting viscosity approach to the Riemann problem for a van der Waals fluid, to appear Archive for Rational Mechanics and Analysis.

L. A. Segel and M. Slemrod, A study of the quasi-state assumption, to be submitted to SIAM Review.

M. Slemrod, Admissibility criteria for phase boundaries, Proc. of Conference on Hyperbolic Problems, St. Etienne, France, ed. C. Carrasso, Springer-Lecture Notes in Mathematics, (1987).

M. Slemrod, The LaSalle Invariance Principle in Infinite Dimensions, Proc. Engineering Foundation Conference on Nonlinear Dynamics, to appear.

M. Slemrod, Recent trends in the continuum theory of phase transitions, in Advances in Multiphase Flow and Related Problems, ed. G. Papanicalaou, SIAM Publications, (1986).

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